TOPOLOGICAL PHENOMENA IN NORMAL METALS *

S.P. Novikov† and A.Ya. Maltsev

L.D. Landau Institute for Theoretical Physics, Ul. Kosygina 2, Moscow 117940, Russian Federation, e-mail: novikov@itp.ac.ru , maltsev@itp.ac.ru

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1. Introduction. Historical remarks.

Measurements of conductivity in the single crystal normal metals in the strong magnetic fields lead to enormous number of different phenomena. Many years ago (about 1956) I.M.Lifshitz, M.Ya.Azbel and M.I.Kaganov formulated following principle:

All such halvanomagnetic phenomena should follow from the geometry of semiclassical electron orbits, based on the one-particle Bloch dispersion relations $\epsilon_n(\mathbf{p})$ only ("Geometrical strong magnetic field limit") - see [1].

Essential new features were found by I.M.Lifshitz and V.G.Peschansky [2]- [3] - details see below. Important experimental results were obtained in the works [4]- [7]. Final book describing results of this period was published

*The main result of this work has been published in [24]. The present article is the extended lecture of authors in the International Conference "Problems of Condensed Matter Theory" dedicated to the 80th birthday of I.M.Lifshitz, Moscow State University, June 1-4, 1997. Its russian variant submitted in Uspekhi Phys. Nauk.

†Also at the Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742-2431, e-mail: novikov@ipst.umd.edu

(see [8]- [10]). A lot of concrete work was done since that, but general understanding of this picture remained unchanged ([8]- [11]).

About 1982 one of the present authors realized that this picture leads to some beautiful problems of the low-dimensional topology ([12], see also [13]- [15]). After that his pupils A.Zorich, I.Dynnikov and S.Tsarev performed purely topological investigations. They finally proved deep theorems, describing topology of the generic open orbits and found geometrical constructions for some very nontrivial nongeneric (”ergodic”) orbits (see [16]- [23]).

Using these topological results, the present authors found universal topological phenomena observable in the conductivity of single crystal normal metals with ”topologically complicated” Fermi surface in strong magnetic field [24] - see below.

Let us point out that some noble metals have topologically nontrivial Fermi surfaces. The first example of this kind was Cu: its Fermi surface was found by Pippard in [25]; Au, Pb, Pt, Ag belong also to this class. There are many other examples now.

We concentrate our attention here on the results of [24] in the theory of normal metals. Using new topological results of [20]- [22] it was observed that these ideas can be applied also in the theory of semiconductors (details see in [26]).

2. Observable Quantities. Generic case.

Let us consider any single crystal normal metal with lattice L generated by the vectors $\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3$. As everybody knows, in the absence of magnetic field $\mathbf{B} = 0$ one-particle electron states can be described by the so-called quasimomenta $\mathbf{p} = (p_1, p_2, p_3)$ defined modulo reciprocal lattice vectors:

$$\mathbf{p} \text{ is equivalent to } \mathbf{p} + \mathbf{l}^*$$

for any vector $\mathbf{l}^*$ such that $\langle \mathbf{l}^*, \mathbf{l}_j \rangle = 2\pi\hbar n_j$, where every n_j is an integer. The reciprocal lattice L^* is generated by the vectors $(\mathbf{l}^*{}_1, \mathbf{l}^*{}_2, \mathbf{l}^*{}_3)$, such that $\langle \mathbf{l}^*{}_i, \mathbf{l}_j \rangle = 2\pi\hbar\delta_{ij}$.

In this approximation we have union of the ”dispersion relations”

$$\epsilon_n(\mathbf{p}) = \epsilon_n(\mathbf{p} + \mathbf{l}^*), \quad n = 0, 1, 2, \ldots, \qquad (1)$$

describing dependence of the energy of electron on the quasimomenta. In standard model electrons in the groundstate occupy all levels below Fermi energy ϵ_F : $\epsilon_j(\mathbf{p}) \leq \epsilon_F$; All higher levels are empty. Conductivity theory in normal metals deals with small perturbations of this picture. All essential phenomena depends on the small neighborhood of the Fermi surface

Most known metals satisfy to the following nondegeneracy conditions:

a) There are no singular points of the dispersion relations on the Fermi level:

$$\nabla\epsilon_j(\mathbf{p}) \neq 0 \quad for \quad \epsilon = \epsilon_F.$$

b)[1] Two different Fermi surfaces correspondent to different branches $\epsilon_j(\mathbf{p})$ and $\epsilon_i(\mathbf{p})$ do not intersect each other: on the Fermi level $\epsilon = \epsilon_F$

$$\epsilon_j(\mathbf{p}) \neq \epsilon_i(\mathbf{p}), \;\; i \neq j$$

Let us describe the most important universal (under the nondegeneracy conditions above) integer - valued observable topological quantities for the conductivity in the strong magnetic fields following results of the work [24]. Apply strong magnetic field $\mathbf{B}$ (of the strength approximately $B \simeq 10T$) [2] and weak electric field $\mathbf{E}$ orthogonal to $\mathbf{B}$. According to our results ([24]), there are two possibilities only:

Case 1. (Compact orbits)

2-dimensional part of conductivity tensor $\sigma_{\mathbf{B}}^{\alpha\beta}$ tends to zero for $B \to \infty$, $\mathbf{B}/B$ fixed, (for $B \sim 10T$ it is already very small): $\sigma_{\mathbf{B}}^{\alpha\beta} \to 0, \alpha, \beta = 1, 2$ in the plane orthogonal to $\mathbf{B}$.

Case 2. (Generic open orbits)

For some direction $\mathbf{B}/B = \mathbf{n}$ the 2-dimensional part of conductivity tensor $\sigma_{\mathbf{B}}^{\alpha\beta}, \alpha, \beta = 1, 2,$ tends to the nonzero constant tensor $\sigma_{\infty}^{\alpha\beta}$ for $B \to \infty$, depending on the direction of the unit vector $\mathbf{n}$. In this case (2×2)-tensor $\sigma_{\infty}^{\alpha\beta}$ has always rank equal to 1: one of its eigenvalues is zero. For the

[1] This property can be destroyed in some metals in strong magnetic fields because of the magnetic breakthrough. See explanations at the end of Chapter 3.

[2] As can be extracted from standard consideration of these fenomena (see for example [8]- [11]), there is the only restriction on the strength of magnetic field: $\omega_B\tau \gg 1$, where ω_B is Larmour frequency, τ - free electron motion time, under which we can observe our "geometrical limit". This condition leads to the magnitudes of magnetic field $\sim 1T$ for pure gold samples at temperatures $\sim 4K$ used in the work [7]. Another restriction for the quasiclassical motion of electron is $\hbar\omega_B \ll \epsilon_F$, but it is valid for all admissible magnitudes of B (the upper bound is of order of $10^3 \sim 10^4 T$).

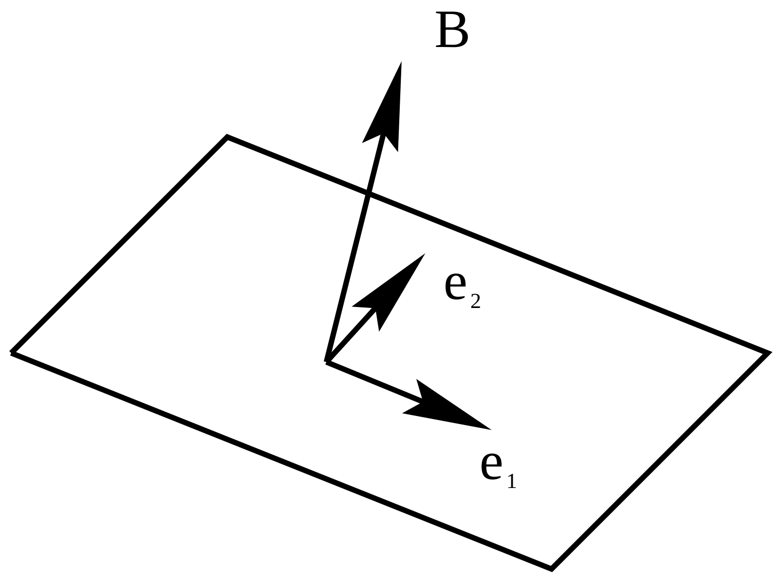

Figure 1: Special basis corresponding to the Case 2.

description of whole 3×3 conductivity tensor $\sigma^{ij}_{\mathbf{B}}, i, j = 1, 2, 3$ we introduce orthogonal basis with first vector $\mathbf{e}_1$ directed along 0-eigenvector of the (2×2)-tensor $\sigma^{\alpha\beta}_{\mathbf{B}}$ in the plane orthogonal to $\mathbf{B}$, second vector $\mathbf{e}_2$ in the same plane $\mathbf{e}_2 \perp \mathbf{B}, \mathbf{e}_2 \perp \mathbf{e}_1$, $\mathbf{e}_3 = \mathbf{B}/B$ (see Fig.1).

We have

$$\sigma^{ij}_{\mathbf{B}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix} + O(B^{-1}) \quad (2)$$

where $(*)$ mean some nonzero constants. In particular, $\sigma^{ij}_{\mathbf{B}} = \sigma^{ji}_{-\mathbf{B}}$ and $\sigma(\mathbf{e}_1) = O(B^{-1})$.

This picture is locally stable: for the magnetic fields with directions $\mathbf{e}'_3 = \mathbf{B}'/B'$ very closed to the original one $\mathbf{e}_3 = \mathbf{B}/B$ we have conductivity tensor of the same structure as (2) in the new orthonormal basis $(\mathbf{e}'_1, \mathbf{e}'_2, \mathbf{e}'_3)$, where $\sigma_{\mathbf{B}'}(\mathbf{e}'_1) = 0,\ \sigma'_\infty \neq 0$ in the plane orthogonal to $\mathbf{B}'$.

Our most important statement is that the plane spanned by the 0-vectors $\mathbf{e}_1$ and $\mathbf{e}'_1$ is integral;

it is the same for all small rotations $\mathbf{B}'$ of the magnetic field $\mathbf{B}$.

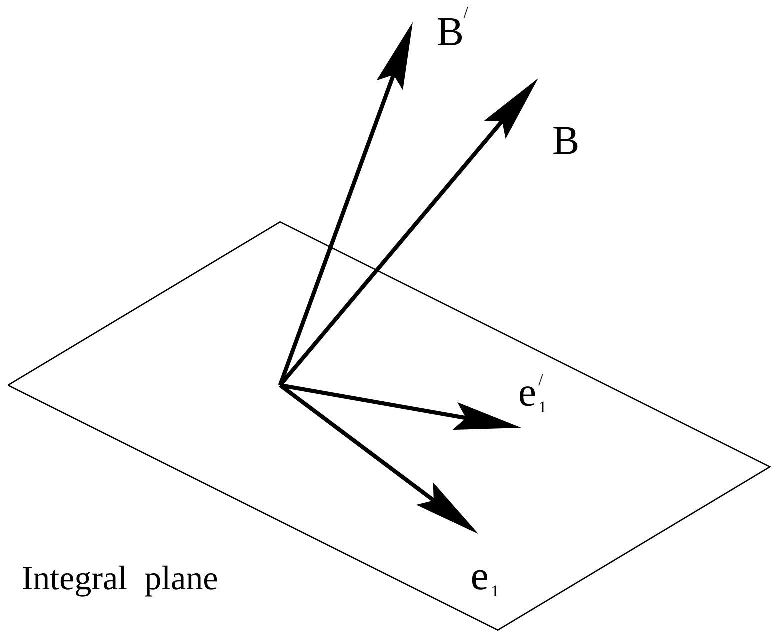

Figure 2: Integral plane generated by vectors $\mathbf{e}_1$ and $\mathbf{e}_1'$.

Integrality means precisely that this plane is generated by two reciprocal lattice vectors $(\bar{\mathbf{l}}^*, \bar{\bar{\mathbf{l}}}^*)$:

$$\bar{\mathbf{l}}^* = n_1\mathbf{l}^*{}_1 + n_2\mathbf{l}^*{}_2 + n_3\mathbf{l}^*{}_3$$

$$\bar{\bar{\mathbf{l}}}^* = m_1\mathbf{l}^*{}_1 + m_2\mathbf{l}_2^* + m_3\mathbf{l}^*{}_3$$

and

$$\mathbf{e}_1 = \alpha\bar{\mathbf{l}}^* + \beta\bar{\bar{\mathbf{l}}}^*, \;\; \mathbf{e}_1' = \alpha'\bar{\mathbf{l}}^* + \beta'\bar{\bar{\mathbf{l}}}^*.$$

Here n_j, m_j - are the integer numbers. Components of the vector $\bar{\mathbf{l}}^* \times \bar{\bar{\mathbf{l}}}^*$ characterize this plane. We are coming to triple of the integers: $n_1m_2 - m_1n_2 = M_3,\; n_2m_3 - m_2n_3 = M_1,\; n_3m_1 - m_3n_1 = M_2$; they defined up to the common multiplier, so the invariant quantities are their ratios in fact.

We call this triple of integers (M_1, M_2, M_3) up to common multiplier the "Topological Type" of conductivity tensor in strong magnetic field $\mathbf{B}$ defined by the pair (or more) magnetic fields $\mathbf{B}, \mathbf{B}'$ with the direction of $\mathbf{B}'$ very close to $\mathbf{B}$ on the unit sphere. Topological type is locally stable on the unit sphere. After any small rotation of the magnetic field topological type

(M_1, M_2, M_3) remains unchanged. Therefore it is the same on some open set of the directions on the unit sphere. We call this open set a "Stability Zone" of the type (M_1, M_2, M_3).

The area (or measure) of the Stability Zone of type (M_1, M_2, M_3) on the unit sphere we shall denote $\mu(M_1, M_2, M_3)$. The area of the zone corresponding to the Case 1 (above) we shall denote μ_0. We have following result

$$\mu_0 + \sum_{(M_1,M_2,M_3)} \mu(M_1, M_2, M_3) = 4\pi \tag{3}$$

(sum along all possible Topological Types). For many Topological Types, in fact, we have: $\mu(M_1, M_2, M_3) = 0$. Anyway, the Topological Types with big enough integer numbers $|M_j|$ correspond a very small value of this area μ.

Therefore in real experiment we can observe only finite (not big) number of Topological Types and their Stability Zones. Mathematically, the equality (3) means precisely that all nongeneric possibilities (i.e. different from Cases 1 and 2) correspond to the directions of $\mathbf{B}$ covering set of zero measure on the unit sphere. Some especially interesting nongeneric pictures will be discussed later.

For the comparison with old experimental data we give here the asymptotic form of the resistance tensor, inverse to σ: $R = \sigma^{-1}$ in the same basis as σ above (2) (see [10], [11]).

Case 1. Order of magnitude of $\hat{R}$ is:

$$\hat{R} \simeq \frac{m^*}{ne^2\tau} \begin{pmatrix} 1 & \omega_B\tau & 1 \\ \omega_B\tau & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{4}$$

(Part of matrix proportional to B is skew-symmetrical).

Case 2. Order of magnitude of $\hat{R}$ is:

$$\hat{R} \simeq \frac{m^*}{ne^2\tau} \begin{pmatrix} (\omega_B\tau)^2 & \omega_B\tau & \omega_B\tau \\ \omega_B\tau & 1 & 1 \\ \omega_B\tau & 1 & 1 \end{pmatrix} \tag{5}$$

here $\omega_B = eB/m^*c$ - Larmour frequency, and τ is free motion time of electrons.

Let us present here some experimental data obtained by Yu.P.Gaidukov [7] for Au. As we can see from (5), we should observe B^2 - dependence of

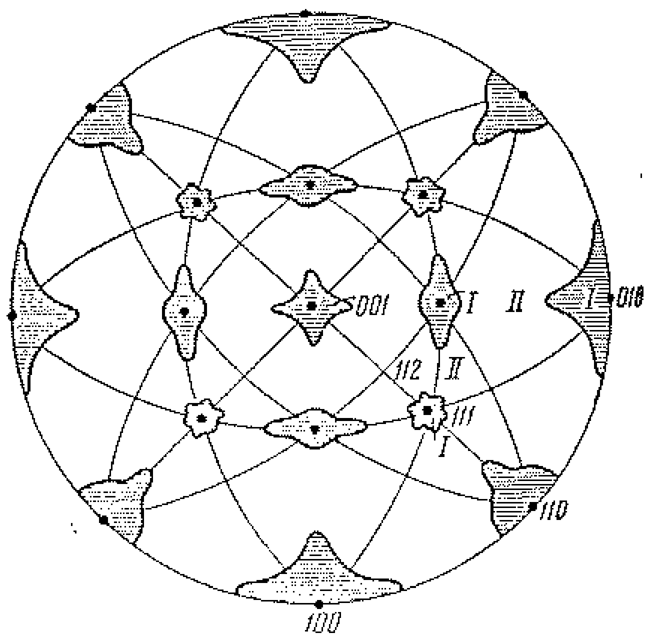

Figure 3: Experimental data obtained by Yu.P.Gaidukov for Au. Black domains correspond to Case 2.

the resistance $\rho \sim (B^2 cos^2\alpha)\rho_0$ in the plane orthogonal to $\mathbf{B}$ in the Case 2. Here $\rho_0 = m^*/ne^2\tau$. The coefficient $(cos^2\alpha)$ is equal to 1 for the electric field directed along the vector $\mathbf{e}_1$ (above) - zero eigenvector for conductivity tensor (2) in the plane orthogonal to $\mathbf{B}$.

On the Fig.3 (Fig.11 in [7]) we can see series of black domains where B^2 - dependence has been observed (1,0,0 means here the direction of vector $\mathbf{B}$, for example).

It is interesting to point out that in the centers of the black domains we find dots where resistance has "very deep minima" and correspond to the Case 1, according to the results of [7]. The resistance within the black

domains should be B^2 - type, like in the Case 2. However, this dependence was found experimentally like B^α for $\alpha \leq 2$ ("slightly less", as written in [7]). Probably, magnetic field $B \sim 1T$ was not enough for our asymptotic behavior. These experiments should be repeated with $B \geq 10T$. In the white area we have the Case 1. Something interesting was found along the black lines (nongeneric situation ?). We shall discuss it later.

Coming back to the interiors of black areas (with central dots removed), we expect here final experimental confirmation of the B^2 - dependence. In this case we can definitely state that these black domains are really the "Stability Zones" whose Topological Types correspond to the integral planes orthogonal to unit vectors directed into the removed central dots: so in the Fig.11 of [7] - our Fig.3 - we can see the following Topological Types:

$$(M_1, M_2, M_3) = (\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1), (\pm 1, \pm 1, 0),$$
$$(\pm 1, 0, \pm 1), (0, \pm 1, \pm 1), (\pm 1, \pm 1, \pm 1). \qquad (6)$$

However, this statement should be verified experimentally.

Until now we consider the results of the important work [7] as a strong experimental evidence that all black domains and some points on the black lines do not belong to the Case 1. Our "integral planes" were not known at that time: nobody asked about them.

Anyway, disappearance of conductivity in the centers of black domains is in good agreement with our understanding of this situation (Au): if our integral plane is orthogonal to magnetic field $\mathbf{B}$, and the generic open trajectories definitely exist for all magnetic fields with directions closed to this one, they really can disappear for this specific field $\mathbf{B}$ (see later).

Case 3. Nongeneric situation (ergodic orbits).

Experimental results described in [7] show some strange behavior of the resistance tensor along black lines on the Fig.3 : in many points of these lines resistance has very deep maxima where asymptotic behavior like B^α has been observed for different values of α: $1 < \alpha < 1.8$. In our opinion, these experimental results should be improved for $B \geq 10T$ instead of $B \sim 2T$ like in [7]. We may expect here much more smaller stability zones of the generic Case 2 with more complicated Topological Types (M_1, M_2, M_3) then (6), but there is also another possibility:

We may expect here more complicated nongeneric "ergodic" orbits of the types discovered in [20]- [23]. By the conjecture of the second author of

this work this situation "typically" leads to the scaling behavior of resistance $R \sim B^{\alpha}, 1 < \alpha < 2$ (see [27]).

Anyway, some examples (γ) of ergodic orbits give such contribution in the asymptotic behavior of 3×3 - conductivity tensor that:

$$\sigma^{ij}_{\mathbf{B}}(\gamma) \rightarrow 0, \tag{7}$$

for $B \rightarrow \infty$ for all $i, j = 1, 2, 3$.

However we must add also contribution of all compact orbits. By the geometrical reasons, we always have some compact orbits in the case of Au (for any direction of magnetic field). Therefore we shall always have nonzero strong magnetic field limit for the conductivity component $\sigma^{zz}_{\mathbf{B}}$ along the magnetic field $\mathbf{B}$

$$\sigma^{zz}_{\mathbf{B}} \rightarrow \sigma^{zz}_{\infty} \neq 0.$$

We expect to have much smaller value of σ^{zz}_{∞} for such nongeneric directions of $\mathbf{B}$ on the unit sphere where ergodic orbits appear (in the comparison with generic neighboring directions). We should see local minima in such points of unit sphere. The last property can be used to distinguish ergodic orbits from small stability zones. There is no reasons to expect any local minima of σ^{zz}_{∞} for small stability zones and strong values of B are needed for the observation of B^2-dependence of resistance in $\Pi(\mathbf{B})$. More detailed discussion of the ergodic orbits see in [27].

3. Topological problems and explanations for the generic case.

We remind here that physical quasimomenta are presented by the vectors $\mathbf{p} = (p_1, p_2, p_3)$ defined modulo reciprocal lattice vectors. From the topological point of view such equivalence classes can be considered as points in the 3 - torus T^3, the "Brillouen Zone". All $\mathbf{p}$ - space R^3 we call an "Extended Brillouen Zone". For topologist it is a "universal covering" over 3 - torus.

In the magnetic field $\mathbf{B}$ we use standard semiclassical description of galvanomagnetic phenomena. The "electron orbits" for the slow time evolution of the electron Bloch waves can be obtained from dynamical system in $(\mathbf{x}, \mathbf{p})$ - space:

$$\dot{\mathbf{x}} = \{\mathbf{x}, \epsilon(\mathbf{p})\} \tag{8}$$

$$\dot{\mathbf{p}} = \{\mathbf{p}, \epsilon(\mathbf{p})\} \tag{9}$$

Here $\epsilon(\mathbf{p})$ is the dispersion relation in the absence of $\mathbf{B}$. Poisson brackets have following form:

$$\{p_i, x_j\} = \delta_{ij}, \ \{x_i, x_j\} = 0,$$

$$\{p_i, p_j\} = \frac{e}{c} \ \epsilon_{ijk} \ B_k \tag{10}$$

For the homogeneous magnetic field $\mathbf{B}$ our equations (9) for the variables (p_1, p_2, p_3) are closed because $B_k = const$. We are coming to the hamiltonian system on the 3 - torus (Brillouen Zone) with Poisson brackets

$$\{p_i, p_j\} = (e/c)\epsilon_{ijk}B_k$$

and Hamiltonian $\epsilon(\mathbf{p})$. It has 2 integrals of motion: $\epsilon(\mathbf{p})$ and $\sum B_k p_k$. The second integral is a "Casimir" for this Bracket in the $\mathbf{p}$-space, because $\epsilon_{jqk} = -\epsilon_{jkq}$:

$$\{p_j, \sum B_q p_q\} = e/c \sum_{q,k} \epsilon_{iqk} B_q B_k = 0$$

Therefore our electron orbits can be represented by 2 equations:

$$\epsilon(\mathbf{p}) = \epsilon_F, \ \ \sum B_k p_k = const. \tag{11}$$

Geometrically, they are sections of the Fermi surface by the planes orthogonal to magnetic field (every plane section is a union of electron orbits).

We call electron orbit compact if it is closed curve in the space R^3 (in the extended Brillouen Zone). We call curve in R^3 periodic with period T and noncompact if $\mathbf{p}(t+T) = \mathbf{p}(t) + \mathbf{l}^*$, where $\mathbf{l}^*$ is some nonzero reciprocal lattice vector. Strictly speaking, this curve is closed in the 3-torus T^3 (Brillouen Zone); topologist will say that such curve in the 3-torus is "nonhomotopic to zero". Compact curves are such that $\mathbf{l}^* = 0$. Topologist will say that they are homotopic to zero in T^3.

We can easily see that periodic noncompact electron orbit may appear only for the magnetic fields $\mathbf{B}$ such that the orthogonal plane $\Pi(\mathbf{B})$ contains at least one vector $\mathbf{l}^* \neq 0$, belonging to the reciprocal lattice.

Let us concentrate our attention on the generic "irrational" magnetic fields $\mathbf{B}$ satisfying to the following restrictions:

1) The plane $\Pi(\mathbf{B})$ does not contain reciprocal lattice vectors.

2) All tangent points of $\Pi(\mathbf{B})$ and Fermi surface are nondegenerate. These points are critical points for the dynamical system (9) on the Fermi surface.

3)Separatrice of the saddle should never come to another saddle for this dynamical system: it should have no second end at all or should come back to the same saddle. Generically there is at most one saddle on any plane parallel to $\Pi(\mathbf{B})$ (orthogonal to $\mathbf{B}$).

Let us introduce a Topological rank of Fermi surface. The equation

$$\epsilon_n(\mathbf{p}) = \epsilon_F$$

in the space of quasimomenta R^3 (Extended Brillouen Zone) is given by the periodic function $\epsilon_n(\mathbf{p})$. This surface in the space R^3 is a union of "connected components", on which any pair of points can be connected by path on Fermi surface. We call Fermi surface "Topologically complicated" if there is at least one connected component (piece) of it which does not lie between any pair of parallel planes. We say that such piece of Fermi surface has topological rank 3 (see Fig.4,a).

We say that Fermi surface has topological rank 2 if any its connected component lies between some pair of parallel planes but there is one which cannot be confined in any cylinder. This piece is like "warped plane with holes" (Fig.4,b). It is possible that there is some second piece (or more) of rank equal 2 with different direction of parallel planes (see Fig.5).

Fermi surface has topological rank 1 if any its connected component can be confined in some cylinder, but there is one which cannot be confined in the box of finite size. This component is like "warped cylinder" (Fig.4,c).

Fermi surface has rank 0 if any its connected component can be confined in some box of finite size (Fig.4,d).

Applying magnetic field we get electron orbits on the Fermi surface as the intersection of the planes $\Pi(\mathbf{B})$ with it. Following topological types are possible:

1. Fermi surface of the topological rank 0. All electron orbits are compact.

2. Fermi surface of topological rank 1. Electron orbits can be compact and open. Open orbits can exist only in the case when magnetic field $\mathbf{B}$ is orthogonal to axis of the warped cylinder on the corresponding piece. But even in this case all orbits can be compact (for example for the "helix" - see Fig.6).

These open orbits (if they exist) should be periodic with period-vector directed along the axis. This picture is obviously nongeneric: open orbits may correspond only to the 1-dimesional family of directions of $\mathbf{B}$ on the unit sphere.

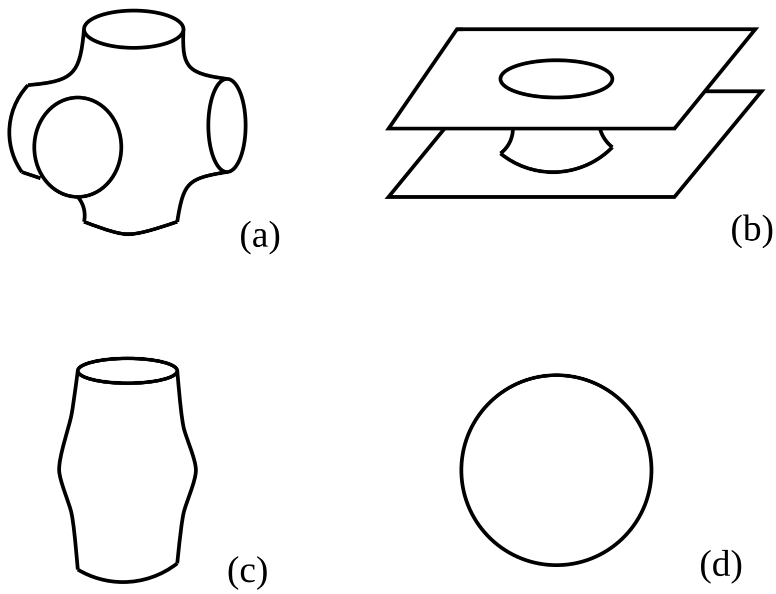

Figure 4: Examples of Fermi surfaces of Topological Rank 3, 2, 1 and 0 respectively.

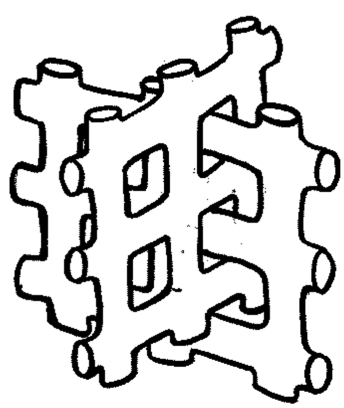

Figure 5: Example of Fermi surface of Topological Rank 2 containing two components with different directions.

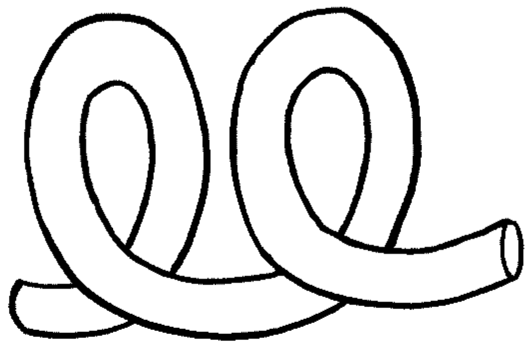

Figure 6: Fermi surface of "helix" type. There is no open orbits for any direction of $\mathbf{B}$.

3. Fermi surface of the topological rank 2. We can have compact and open electron orbits here for any direction of magnetic field $\mathbf{B}$.

As was said above, generally speaking, different connected components of our Fermi surface may be confined between the parallel planes with different (integral) directions. All open orbits, if they exist, obviously have mean directions given by the intersection of $\Pi(\mathbf{B})$ with integral planes $\Pi_j(\epsilon_F)$ of the corresponding connected components. Integrality of these planes can be easily derived from the periodicity of Fermi surface. If the direction of $\mathbf{B}$ is irrational ($\Pi(\mathbf{B})$ does not contains reciprocal lattice vectors), open orbits can exist on the components of rank 2 corresponding to one integral direction only (we imply here that Fermi surface is nonselfintersecting). Therefore all open trajectories have the same mean direction given by the intersection of some integral plane Γ with $\Pi(\mathbf{B})$. This picture is locally stable under small rotations of $\mathbf{B}$ and plane Γ can be observed experimentally.

This picture represents already the situation corresponding to the generic Case 2 (above) for the conductivity tensor. Topological Type here should correspond to the very same integral planes $\Pi_j(\epsilon_F)$, described by the integers (M_1^j, M_2^j, M_3^j) for all $\mathbf{B}$ where open orbits exist (except the case $\Pi_j(\mathbf{B}) =$

$\Pi(\epsilon_F)$. For the nongeneric case $\Pi(\mathbf{B}) = \Pi(\epsilon_F)$ all open orbits (if they exist) are periodic in all components.

4. Consider now the most interesting case of Fermi surface with Topological rank 3. We are dealing in fact with one piece of the topological rank 3 only. Identifying the equivalent points in the space of quasimomenta

$$\mathbf{p} \equiv \mathbf{p} + \mathbf{l}^*$$

modulo reciprocal lattice vectors, we get a closed 2-dimensional manifold (surface) in the 3-torus - "Brillouen Zone". Consider now "fully irrational" magnetic field $\mathbf{B} = (B_1, B_2, B_3)$ such that $\Pi(\mathbf{B})$ does not contain integral (i.e. reciprocal lattice) vectors $\mathbf{l}^*$ and corresponding family of electron orbits satisfies to the nondegeneracy conditions 1,2,3 (above).

Remove from Fermi surface all nonseparatrix compact orbits (all of them are closed curves in the space of quasimomenta R^3). Remaining part is obviously a union of surfaces, whose boundaries are the separatrices:

Fermi surface minus all nonsingular compact orbits = union of pieces S_i (if there are open orbits). Boundary of S_i is a set of some closed separatrix curves $\gamma_{i\alpha}$ (see Fig.7)

Every closed separatrix curve is a plane curve in $\Pi(\mathbf{B})$. Its interior is a topological 2-disc $D_{i\alpha}$. Fill in all boundaries $\gamma_{i\alpha}$ by the plane 2-discs $D_{i\alpha}$ and add them to the surfaces S_i (partial Fermi surfaces). Finally we get closed 2-surfaces in R^3 (and their images $\bar{S}_i$ in the 3-torus T^3 after the identification of the equivalent quasimomenta). By definition, all open orbits lie on these pieces.

We call the genus of compact 2-manifold $\bar{S}_i$ in the 3-torus by the "genus of corresponding open orbits", lying on it.

The most important nontrivial topological result, extracted by the authors from the works of A.V.Zorich and I.A.Dynnikov [16]- [22] is that generically all these pieces $\bar{S}_i$ have genus 1; it means that surfaces $\bar{S}_i$ are topologically equivalent to the 2-dimensional tori, imbedded in the Brillouen Zone - 3-torus T^3. The final general proof of this theorem is very nontrivial (see [19]) and we shall not try to present it here. "Generically" means in fact that if this statement is wrong for some energy level $\epsilon(\mathbf{p}) = \epsilon_0$ than for all small nonzero perturbations of energy level $\epsilon(\mathbf{p}) = \epsilon_1$ it will be true. We can easily find out that these surfaces $\bar{S}_i$ are in fact imbedded in T^3 without selfintersection. They do not intersect each other as well. Last property leads

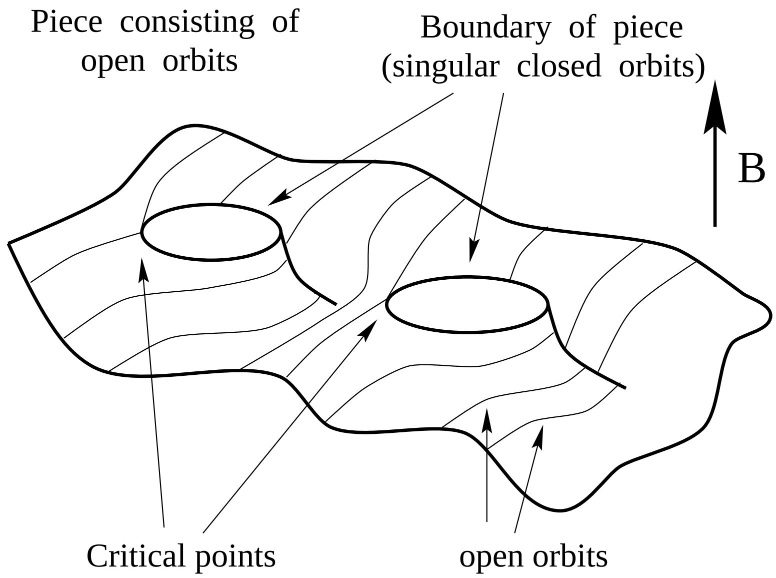

Figure 7: Piece S_i consisting of open orbits. Singular closed orbits $\gamma_{i\alpha}$ with critical points are the boundaries of S_i.

to the following result: every piece S_i in the space of quasimomenta R^3 looks like "warped plane" after filling in by 2-discs $D_{i\alpha}$. Therefore its section by the planes parallel to $\Pi(\mathbf{B})$ lie in the strips of finite width in these planes.

Local stability of this topology follows from the fact that all compact curves in our constructions (like compact orbits and compact separatrices with two ends in the same saddle, which is a limit of compact nonsingular orbits) are stable under the small rotations of the magnetic field. Our non-degeneracy requirements for the metal and "generity" requirements for the direction of $\mathbf{B}$, are both important for this conclusion. From these arguments follows full foundation of the Case 2 for the generic conductivity tensor.

Let us point out that the first example of such topologically stable open orbits was discovered by I.M.Lifshitz and V.G.Peschansky in [2] for the Fermi surface of Cu (or for the so-called "thin" space net, as they called it after Pippard's discovery of Fermi surface for Cu). For this "thin space net" (see Fig.8 - Fig.2-3 in [2]) we can have open orbits correspond to the Case 2 for the conductivity tensor, where topological types may take only following values:

$$(M_1, M_2, M_3) = (\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1).$$

In the case of cubic lattice corresponding thin space net leads to the stability zones of the small areas surrounding the directions above. Other part of unit sphere belongs to the Case 1. It seems that in the second work [3] open orbits were found also for the more complicated Fermi surfaces, but nothing like our integral planes was discussed. The authors of [3], however, thought that they found open domains on the unit sphere where 2 different mean directions of open orbits coexist (see fig.4 of [3] - the last picture on our Fig.9). This result of [3] is mistakable and contradicts to our results. This situation is impossible for the open domains on the unit sphere. Our statement is a mathematically rigorous corollary from the work [19].

In general, topological types of stability zones will be much more complicated even for the same Fermi surfaces.

In the same works [2]- [3] the contribution of the open orbit ot this type in conductivity tensor was calculated. The last result is valid for all open generic trajectories used in our paper [24] and here. Besides that, our results are based on the important additional property, extracted from topology: All generic open orbits for given magnetic field have the same mean direction.

The last property is true for the generic type of the open trajectories only. For example, if magnetic field $\mathbf{B}$ is nongeneric and $\Pi(\mathbf{B})$ is an integral plane, than we may have open orbits of different integral directions $\mathbf{l}^*$. In this case every open trajectory is periodic. Everyone of them gives contribution like in [2], but total sum of "partial conductivity tensors" $\sigma_{i\infty}^{\alpha\beta}$ will have different algebraic structure (for example, no zero eigenvector will be found in the plane $\Pi(\mathbf{B})$). Small rotation of $\mathbf{B}$ destroys this picture. We can see that for the magnetic field $\mathbf{B}$ such that $\Pi(\mathbf{B})$ contains only one integral vector $\mathbf{l}^*$ (up to proportionality) such situation is impossible.

Let us point that it is possible for some symmetry that two different Fermi surfaces (2 connected components in mathematical terminology) can intersect each other in strong magnetic field (see [28], [10]). It means precisely that electrons on each component move completely ignoring other component in the limit $B \to \infty$. Physical criteria for this phenomena (magnetic breakthrough) can be found in [10]. In this case only we can have in principle two different families of open orbits with different mean directions, such that this picture is stable under the rotations of $\mathbf{B}$. Total sum of the contributions of both components in the conductivity tensor will be a sum of the same types of tensors like in the generic Case 2 with different directions of the integral planes. However the Topological Types of partial integral planes have differ-

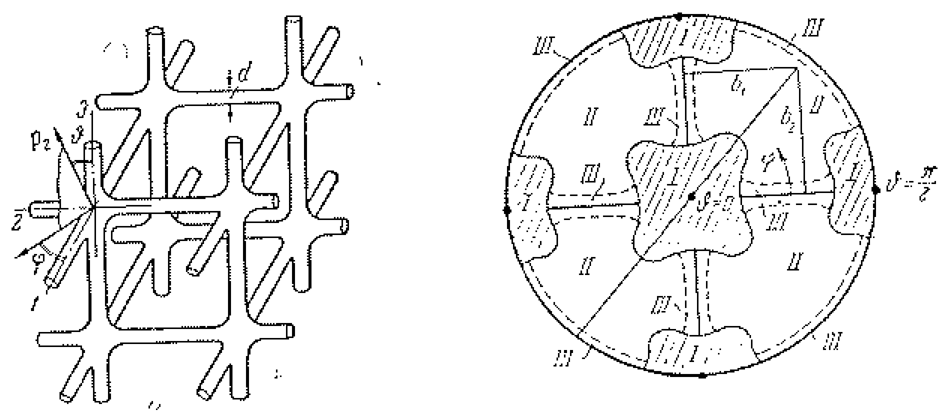

Figure 8: The so-called "Thin space net" and stability zones on the unit sphere corresponding to it. As was pointed by I.M.Lifshitz and V.G.Peschansky, mean directions of open orbits in these stability zones are given by the intersections of $\Pi(\mathbf{B})$ with coordinate planes (xy), (yz), (xz).

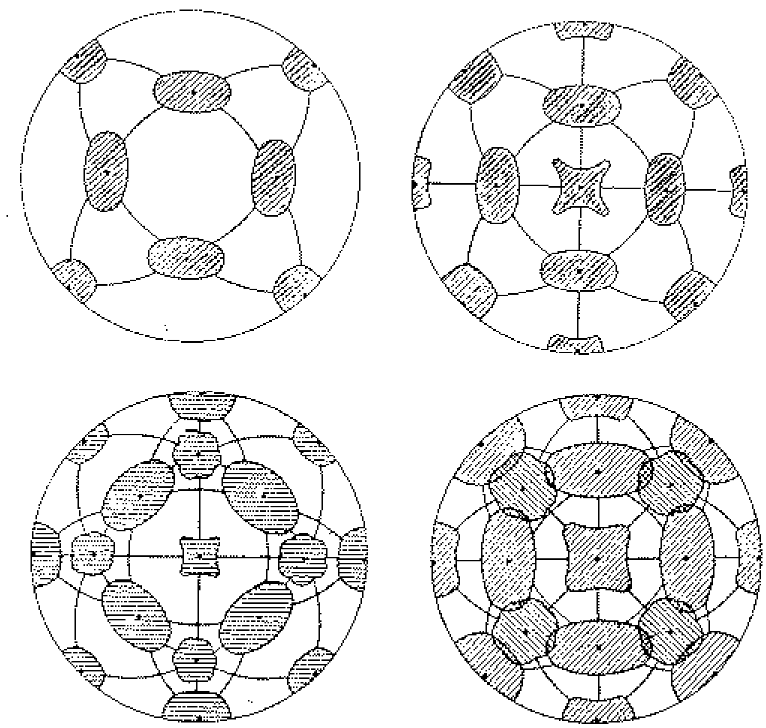

Figure 9: Stability zones represented by I.M.Lifshitz and V.G.Peschansky for different analytical examples of Fermi surface for cubic lattice. For these zones our integral planes were not discussed. According to our results, the last picture is mistakable. Stability zones can not intersect each other on the whole domain on the unit sphere and open trajectories with different mean directions can not exist for the whole domain of nonzero measure on the unit sphere.

ent stability zones on the unit sphere. Therefore rotating magnetic field we may observe their intersections, where we can see the case more complicated then the Case 2. (M.I.Kaganov was the first who pointed out to us on the magnetic breakthrough problem during the authors lecture.)

Consider now the nongeneric Case 3 of ergodic orbits. They are unstable under the small rotation of the direction of **B** on the unit sphere. Let us remind that the Cases 1 and 2 are "generic" in the sense that they correspond to the open everywhere dense domain on the unit sphere. Using some additional arguments (extracted from [22]) we are coming to the conclusion that measure of the set of directions corresponding to the ergodic orbits is equal to zero. Its Hausdorf dimension is unknown.

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